SIMPLE METHODS FOR PREDICTING THERMAL BEHAVIOUR AND ENERGY CONSUMPTION IN BUILDINGS

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1. INTRODUCTION

Today's designing needs call for an ever greater evaluation and comparison of different solutions to determine the optimum values of parameters such as glass surfaces, orientation, type of insulation etc., so as to obtain a better building thermal behaviour. These evaluations do not only deal with energy requirements, but also with room temperatures existing in non air-conditioned rooms or with thermal loads in relation to given outside conditions. Mathematical models have been recently set up to enable the detailed simulation, by computer, of the thermal behaviour of buildings undergoing real meteorological conditions.

These models, however, are not readily available and their use is so far difficult and expensive. Nevertheless, they permit easily the exploration of a great number of building types and of climatic conditions, thus determining the influence of the different parameters on the building's thermal balance. They are therefore particularly suitable to obtain the data necessary to establish simplified calculation methods for energy requirements for the summer and winter climatization; such methods can be so simple as to be worked out manually or by small desk computers on the basis of only a few synthetic climatic data. This methodology has been adopted to obtain the algorithms here submitted, the determination of which was set up as per NBSLD computer program /1/.

2. A SIMPLIFIED METHOD FOR THE ESTIMATION OF ENERGY CONSUMPTION

2.1 Required input data

The following algorithms permit the evaluation of energy needs for the heating and/or cooling of a building, the inside temperature of which is set within a specified temperature span by an appropriate system. The building's energy need is evaluated on the assumption that the plant capacity is always sufficient to maintain the temperature within these limits. To apply the procedure here suggested the following data are needed:

A) Building's data
- Dimensions and orientation
- Thermophysical properties of outer and inner walls.

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B) Working conditions in the considered time span
- -Set-point inside temperature for the heating and/or cooling system
- -Mean number of air changes per time unit
- -Mean internal heat gains (lighting, occupants, etc.).

C) Climatic data for the considered and previous time spans
- -Mean outside temperature
- -Total solar radiation falling on differently oriented surfaces
- -Mean wind velocity.

These climatic data are easily obtainable from statistical evaluations already available for several places.

2.2 Building's thermal balance

In the considered P time span, for which the energy requirement has to be evaluated, one can list s quantities of energy E_i (i=1, s), going in and out the building; each of them will be given a sign, the outgoing quantities being assumed as positive. As is shown later, in setting the building thermal balance, one can demonstrate that the heating or cooling system energy requirements cannot be taken as the algebraic sum of the above quantities, in as much as some phenomena, due to the variable outside thermal conditions and to the building's thermophysical characteristics occur. In the following pages, after having examined in detail the various components E_i, a calculation procedure (called "SMECC") of the above phenomena will be offered for the evaluation of the energy requirements.

2.2.1 Heat amount transmitted through the envelope and caused by different outside and inside air temperature. E_W is the amount of heat passing through the building's envelope (opaque walls and windows) following directly the temperature difference between the inside and the outside of the building. Being this difference variable and supposing for now a constant energy level of the envelope (this phenomenon will be studied in par. 2.2.2), the total energy going through the envelope can be evaluated as if conditions were stationary; the difference of temperature between the inside and outside must be duly integrated. The energy quantity going through the envelope can be expressed:

$$E_W = f_c \sum_{1}^{d} {}_j\, U_j\, S_j \cdot \int_0^P (t_a - t_e)\, d\tau \qquad (1)$$

where:

d: number of dispersing surfaces

f_c : reduction coefficient, defined below

t_a value will be equal to the set thermostat lower temperature $t_{a,min}$ when it comes to evaluating heating requirements, or to the upper temperature $t_{a,max}$ if cooling requirements have to be determined. If T represents the day span and N=P/T the number of days in the considered period P, the integral of eq. (1) can be:

$$\int_0^P (t_a - t_e)\, d\tau = \sum_{1}^{N} {}_i \left[\int_{iT}^{(i+1)T} (t_a - t_e)\, d\tau \right] \qquad (2)$$

Being $t_{em,i}$ the mean outside temperature of the day, one obtains:

$$\int_0^P (t_a - t_e)\, d\tau = T \sum_{1}^{N} {}_i\, (t_a - t_{em})_i = T\ D \qquad (3)$$

the quantity D is usually called "degree days" for the considered period; the more widely used method of calculating energy requirements (known as "degree-days method" /2/) is called after this quantity. Such method however allows only the calculation of heat amounts related to the inside and outside air temperature (i.e. transmission through walls and ventilation); solar radiation is not taken into consideration and therefore this method can only be used for winter conditions and even then, although conservative, it is inaccurate and not suitable anyway for the design stage whenever solar radiation cannot be neglected. To consider some of the free heat gains in winter conditions, the "degree-days" usually available are calculated referring to a temperature (17° ÷ 19°C) lower than the value of the room temperature (t_a = 20°C) and the summation is limited only to the days when t_{em} is less than a set temperature (generally 12°C). To avoid confusion with the above "degree-days", reference is made in this paper to a t_{em} temperature, which is the mean in the considered P time span, obtaining as per eq. (3):

$$\int_0^P (t_a - t_e)\, d\tau = T\ N(t_a - t_{em}) \qquad (4)$$

By using this relationship, eq. (1) becomes:

$$E_W = f_c \left(\sum_{1}^{d}{}_j\ U_j\ S_j\right) T\ N\ (t_a - t_{em}) \qquad (5)$$

It is known that the transmittance U of each dispersing surface can be written:

$$U = \frac{1}{\frac{1}{\alpha_e} + \sum_{1}^{u}{}_j \frac{s_j}{\lambda_j} + \frac{1}{\alpha_a}} \qquad (6)$$

where:

u: number of layers of the wall

The surface heat transfer coefficient α_e can be greatly varied by the wind velocity v and by the outside finish of the wall; so the following relationship will be useful to calculate α_e in W/m^2K /3/:

a) rough surfaces $\alpha_e = 10.75 + 1.12\ v$ (7)

b) glass surfaces $\alpha_e = 8.22 + 0.888\ v - 0.0024\ v^2$ (8)

For the surface heat transfer coefficient α_a, when estimating U, the standardized values shown in handbooks and standards are used. It is, however, necessary to observe that for a correct evaluation of the amounts of heat passing through the walls defining the envelope of a room, one must also consider the mutual longwave heat exchange by radiation between the internal surfaces of the same; this influence is revealed by a change in the internal surface heat transfer coefficient depending on the number and type of transmitting surfaces. This phenomenon can be taken into consideration by including the standard value α_a of the internal surface heat transfer coefficient and by applying to the heat amount thus evaluated a suitable correction coefficient f_c which depends only on the room characteristics. This coefficient has been determined by setting up a detailed room thermal balance, so as to consider separately the thermal convection and radiation heat exchanges taking place on the internal wall surfaces, in different conditions /4/. So the following expression has been obtained for f_c (see fig. 1):

$$f_c = 1 - 0.194\ U_m + 0.0207\ U_m^2 \qquad (9)$$

as a function of the mean room transmittance U_m expressed in $W/m^2\,K$ and defined as:

$$U_m = \frac{\sum_{1}^{d}{}_j\ U_j\ S_j}{S_T} \qquad (10)$$

where:

S_T: total internal room area.

The f_c coefficient evaluation, however free from problems in a single room, has to be carefully worked out when it comes to embrace a block of rooms as a whole. In fact, as consideration must be taken of the mutual thermal radiation exchanges between each external wall and the corresponding room at the back, in such a case, it will be necessary to give U_m a suitable mean value in eq. (9).

2.2.2 Energy accumulated in the structures. In any P time span the energy requirement of which has to be evaluated, the final temperature of the structure and therefore the energy accumulated in it is generally different from what it was initially; such quantity of energy has to be taken into account in the building thermal balance. An exact calculation would need to consider the initial and final mean temperatures of each building component in the given time. The available data are instead the mean air temperatures in the given time and in various other spans of time preceding it. Once these boundary conditions for every wall are known, one can estimate the related mean temperatures ($\bar{t}$) between the inside and the outside during every time span. The temperature variations between two antecedent time spans are significant for the considered period depending on the thermal characteristics of the walls. If P_o is the walls "response time" /5/, the total energy variation for the i-th wall during the time span under consideration can be so approximated:

$$\Delta e_i \cong \left(\sum_{1}^{u} {}_j \varrho_j c_j s_j\right) S_i \left\{ (\bar{t}_{-1}-\bar{t}) \left[1-\exp\left(-\frac{P}{P_{o,i}}\right)\right] + (\bar{t}_{-2}-\bar{t}_{-1}) \left[\exp\left(-\frac{P}{P_{o,i}}\right)-\exp\left(-\frac{2P}{P_{o,i}}\right)\right] + \dots \right\} \quad (11)$$

where:

$\bar{t}_{-n}$: mean wall temperature during the n-th antecedent time span.

If $P \gg P_{o,i}$ (and this situation happens for normal structures with P>10 days), eq. (11) becomes:

$$\Delta e_i \cong \left(\sum_{1}^{u} {}_j \varrho_j c_j s_j\right) S_i \; (\bar{t}_{-1}-\bar{t}) \quad (12)$$

The quantity Δe_i is partly exchanged with the internal space, partly with the outside: by resorting to the principle of effects superposition, one can approximate the heat quantity ε_i wholly exchanged with the internal space by the i-th wall, as follows:

$$\varepsilon_i = f_c \; \Delta e_i U_i \; \left(\frac{R_{w,i}}{2} + \frac{1}{\alpha_e}\right) \quad (13)$$

If the building's walls subject to mean temperature variations are z (including partitions and floors, if subject to summer overheating), the total energy exchanged for the considered building shall be:

$$E_M = -\sum_{1}^{z} {}_i \; \varepsilon_i \quad (14)$$

2.2.3 Heat amounts due to ventilation and/or infiltration. The overall quantity of energy due to ventilation and external air infiltration during the period P, can be expressed as:

$$E_V = \int_0^P \varrho c \; V(t_a - t_e) d\tau \quad (15)$$

where:

ϱ : density of outdoor air
c : specific heat of outdoor air
V : volume flow rate of outdoor air

hence, by means of simple developments, similar to what has already been seen in the previous paragraph, one obtains:

$$E_V = \varrho c \; V_m \; (t_a - t_{em}) \; T \quad N \quad (16)$$

where:

V_m : mean flow rate during the considered time span.

2.2.4 Heat amounts due to solar radiation on opaque walls. Solar radiation reaching the opaque external surfaces of the building is partially absorbed by it and therefore adds energy to the building itself. Using the concept of "sol-air" temperature t_{sa}:

$$t_{sa} = t_e + \frac{a}{\alpha_e} q_r \qquad (17)$$

where:

a: hemispherical absorbitance of the exterior surface
q_r: flux of solar radiation falling on that surface

and, as the effect of the outside temperature t_e has already been analysed in par. 2.2.1, it is possible to compute the heat quantity E_R coming into the building by using the principle of effects superposition, like in:

$$E_R = -f_c \sum_{1}^{d}{}_j \left(U_j S_j \frac{a_j}{\alpha_e} I_j \right) \qquad (18)$$

where:

I_j: amount of solar radiation falling per unit of surface in the given period P.

The values of solar radiation falling on differently oriented surfaces can be obtained from the total radiation on horizontal surface by means of the procedure suggested in /6/.

2.2.5 Heat amounts due to solar radiation on glass surfaces. The solar radiation arriving on the building's glass areas is absorbed, reflected outside and transmitted inside according to their optical properties and its angle of incidence. As it is known, the amount of energy reaching the room can be satisfactorily expressed by the product of the ξ "shading coefficient" characteristic of the glass considered /7/, with a value of 0.87 that expresses approximately a single strength glass transmission coefficient. This amount of high frequency radiant energy is absorbed by the internal walls and later given back by convection to the air and by low frequency radiation to the surrounding surfaces. If, according to the principle of effects of superposition, the temperature difference between the outside and inside air is nil, it's easy to take into account the relative temperature rise of the walls and the consequent convection-conduction heat loss towards the outside. Such phenomenon can be estimated by means of a reduction coefficient f_r (see fig. 1) obtained from the room's thermal balance, and it can be shown:

$$f_r = 1 - 0.32 U_m + 0.03 U_m^2 \qquad (19)$$

where U_m (expressed in W/m^2 K) has expression (10).
It is moreoever necessary to take into account another coefficient φ showing the average fraction of the glass area exposed to the sun in the given P time span, including shading from rolling shutters, screens or other buildings. Therefore, if g is the number of windows in the building, the amount of energy for an internal balance shall be:

$$E_F = -0.87 \; f_r \sum_{1}^{g}{}_j \left(\xi_j \; \varphi_j \; S_j I_j \right) \qquad (20)$$

where:

I_j: amount of solar radiation arriving per surface unit on the j-th window in the P span of time
S_j: area of the j-th window

2.2.6 Heat amounts due to solar radiation storage in the buildings structures. As already seen in par. 2.2.2 for the effects of long term outside temperature variations, it is possible to treat similarly also the variations of the mean solar radiation intensity, by estimating the consequent mean temperature variations undergone by the structure. For this purpose, with the help of the superposition of the effects principle and the concept of "sol-air" temperature, one can evaluate, with reference to steady state, the mean temperature ($\bar{t}'$) of the different sunlit walls in contact with air at 0°C on one side and on the other with air at a temperature:

$$\bar{t}' = \frac{a\, q_r}{\alpha} \qquad (21)$$

where:

q_r : mean solar radiation flux per time unit falling on the wall

α : inside or outside surface heat transfer coefficient depending on the position of the considered surface.

The calculation of variations of stored energy should be related to both the radiation arriving to the outside surfaces of the building and the one reaching the internal surfaces of the rooms; in the latter instance a certain distribution of solar radiation arriving through the windows onto the various internal surfaces must be assumed.

In the temperatures $\bar{t}'$, computed by eq. (21) for the different walls in the P time spans, the procedure is altogether similar to the one in par. 2.2.2; in eq. (12) too, and in the same hypotheses, the variation of energy for any i-th wall can be written as:

$$\Delta e'_i \cong \left(\sum_{1}^{u}{}_{j}\; s_j c_j \varrho_j\right) S_i \;(\bar{t}'_{-1} - \bar{t}') \qquad (22)$$

where:

u: number of layers of the wall

From this, along with eq. (14), the total amount E_s of stored solar energy exchanged during the considered time span P can be expressed as:

$$E_S = -f_c \sum_{1}^{y}{}_{i}\; \Delta e'_i \;\; U_i \left(\frac{R_w}{2} + \frac{1}{\alpha}\right) \qquad (23)$$

where:

y: number of considered walls (generally different from the z number shown in eq. (13)).

Conclusions similar to the ones in par. 2.2.2 can be reached when the considered span is compared to the wall's response time.

2.2.7 Amounts of heat due to internal energy sources. Inside buildings one can find sources of energy other than those from air-conditioning system, like for instance the electric system, stoves, fridge's condensers, electrical household appliances; people too can be considered. Heat transmission to the room happens mainly by convection and radiation; having mentioned in par.2.2.5 how radiant energy affects the thermal balance of the building, it becomes clear that radiant components must be treated separately from the convection ones, by resorting precisely also to the f_r coefficient as in eq. (19). Therefore the heat amount due to internal energy sources is expressed by:

$$E_I = -\left[f_r \sum_{1}^{r}{}_{i}\; (W_{rm})_i + \sum_{1}^{c}{}_{j} (W_{cm})_j\right] T\; N \qquad (24)$$

where:

W_{rm} : mean thermal power originated by radiation in the time span P by the internal sources

W_{cm} : mean thermal power originated by convection in the time span P by the internal sources

r : number of radiant sources

c : number of convective sources

2.3 Evaluating energy consumption.

The heat amounts intervening in a building's thermal balance have been covered in the preceding paragraphs. As already seen, the sign of some of them is completely defined, while for others, it is related to the thermal condition to which the building undergoes in the given P time-span. Each of them can perform a pro or against role in regard to energy requirements, whether heating or cooling is studied. One must distinguish also the case when, in a given time span, only the heating or cooling system is working, from the one when both are working in relation to the respective set-points.

a) Energy requirements for heating only If the heating requirements of a building have to be estimated, all the amounts of energy with a positive sign (with an outgoing direction), will form an energy requirement which the system has to provide; Θ indicates their total, so:

$$\Theta = \sum E_i \quad (\forall E_i > 0) \tag{25}$$

Yet in the building's thermal balance, the amount of energy needed is compensated by energy gains which, with a negative sign, have an incoming direction; let Π be their sum, therefore:

$$\Pi = \sum E_j \quad (\forall E_j \leqslant 0) \tag{26}$$

It's easy to realize that the building's net requirements cannot be given simply by their algebraic sums: during the span P, in fact, the building's emerging and entering thermal powers are subject to remarkable variations (only think of the ones originated by solar radiation); there could be some spans of time when the energy gains are abundant in relation to heat losses, thus originating a rise in the internal temperature and the switching off of the system, with dispersions to the outside bigger than the ones evaluated with an inside temperature equal to the set-points. This way the addition of energy Π is not fully utilized, and this must be considered in estimating the consumption C_H by inserting a reduction coefficient η that leads to:

$$C_H = \Theta + \eta \Pi \tag{27}$$

The reduction coefficient η can be expressed, not only as a function of the ratio Θ/Π in the given time between total loss and total gain, but also as a function of the thermophysical characteristics of the building and particularly its thermal inertia and mean transmittance; it is reasonable to assume that a bigger thermal inertia and a greater mean building's transmittance are both entities opposing the overheating caused by the peak loads of energy collected by the building. The mean transmittance can still be represented by the U_m parameter defined in (10), while the thermal inertia can be taken into account by means of the specific mass defined as:

$$M = \left[\sum_{1}^{d}{}_j \; p_j S_j + \frac{1}{2} \sum_{1}^{b}{}_i \; p_i S_i \right] / S_f \tag{28}$$

where:

d : number of dispersing walls

b : number of non dispersing walls

p : mass of every wall, per area unit

S_f : floor area of the room

As already shown in /8/, whenever estimating M, particular attention must be paid to the position of insulation inside the walls, generally assuming that only the interior parts of the walls characterize the thermal inertia of the building. Moreover to estimate energy requirements of a group of rooms (e.g. a whole building), one must select a suitable mean value for M from those of the single rooms.

Numerous tests, using NBSLD computer program, and considering various climatic data and very different buildings, have made it possible to obtain for η the trend reported in figs. 2, 3 and 4 for various values of U_m and different conditions of specific mass M. An approximate analytical expression for η is:

$$\eta = \frac{\Theta/\Pi}{\Theta/\Pi + \varphi} \tag{29}$$

where:

$$\varphi = \left[1.561\ U_m - 0.151\ U_m^2 + 0.168\ \left(\frac{M}{100}\right) - 0.016\ \left(\frac{M}{100}\right)^2 - 0.126\ \left(\frac{M}{100}\right)\ U_m\right] \cdot$$

$$\cdot \exp\left\{-\left[1.58\frac{\Theta}{\Pi} - 0.126\ \left(\frac{\Theta}{\Pi}\right)^2\right]\right\}$$

with the condition that when the resulting value of η is greater than Θ/Π one must assume:

$$\eta = \frac{\Theta}{\Pi} \tag{30}$$

b) Energy requirements for cooling only In this case, contrary to the previous one, the energy requirement consists of the quantity of heat entering (negative sign), while the outflowing energy (positive sign) consists of "free" components in the global balance; therefore this time:

$$\Theta = \sum E_j \quad (\forall E_j \leq 0) \tag{31}$$

$$\Pi = \sum E_i \quad (\forall E_i > 0) \tag{32}$$

Since the phenomenon taking place in the building is largely the same as in the previous case, however reversed, it is possible to use the same methodology, writing again:

$$C_C = \Theta + \eta\Pi \tag{33}$$

where for η the curves of figs. 2, 3 and 4 and expressions (29) and (30) are always valid in relation to Θ/Π and to the characteristics of the building. In this case, in fact, the quantities of heat directed towards the outside are inclined to lower the internal temperature below the set-point expected of an air-conditioning system, reducing E_W.

c) Both cooling and heating required In the intermediary seasons and whenever the set-point values for heating and cooling units are very close, it could happen that in the same considered period of time P, heating and cooling are both required. In this case, therefore, in the sum (27) the quantities of heat C_C supplied by the cooling unit must be added (as a component of Θ), and the same applies in the balance (33) for the quantity of heat C_H supplied by the heating system. The calculation therefore must be carried out by trial and error, calculating for example C_H after having taken a certain C'_C to be included in (27); once C_H is known, one can then calculate C_C through eq. (33); if C_C and C'_C differ excessively the procedure must be repeated with the new value of C_C (27), until convergence is reached. At this point the correct values of C_H and of C_C will be known. It must be pointed out that when the considered situation occurs, the values of C_C and C_H are much lower than those of Θ and Π; particular attention should therefore be paid in evaluating η to avoid relevant percent errors: anyway, these errors have generally a negligible influence on the overall seasonal balances.

2.4 Influence of different parameters on η.

The trend of the coefficient η, as shown in figs. 2, 3 and 4, emphasizes that the actual distribution of the incoming and outgoing quantities of heat taking place in a building acts in such a way that the utilization of the "free energy" is the higher, the higher the ratio Θ/Π, the thermal inertia of the building (specific mass M) and the heat losses are.

By recalling expressions (27) and (33), it can be easily verified that C = 0 only if $\eta = \Theta/\Pi$, while in observing the trend of η it can be noted that even when "free" energy (Π) is slightly higher than the demand (Θ), in many cases $\eta < \Theta/\Pi$, and therefore $C \neq 0$. This fact can be explained taking into account that, even in these conditions, the inevitable time-delay between requirements and available "free" quantities of heat may cause the latter not to be able to cover moment by moment the energy requirements of the building. Furthermore, it should be pointed out that it is possible to have an idea, however very approximate, of the maximum variations occurring in the internal temperature in relation to a set-point temperature; in fact referring, for example, to the heating at a certain set-point temperature, the maximum overheating of the room corresponds to that temperature which, used as set-point for the air-conditioning, in the same period of time, will result in no consumption.

2.5 Comparisons.

The results obtainable with the above shown energy requirement forecast method have been compared, considering P = 1 month, for different climatic and architectural conditions, with those provided by the complete NBSLD hourly simulation computer program. An example of this comparison is given in fig. 5. The reasonable agreement between the two methods has nevertheless brought forth the importance of evaluating the storage of energy in the structures whatever the span P examined. Yet besides the simplicity of applying this method and the easy accessibility of the climatic data required, it is possible to obtain very detailed information on the influence exercised on energy consumption by the various morphological and constructive parameters of the building, thus allowing an evaluation of the benefit obtained from certain decisions, both in the planning and in the "retrofitting" stage.

3. THERMAL LOAD AND INTERNAL TEMPERATURE EVALUATION

A careful study of buildings on "design days" is necessary both for the correct dimensioning of climatization systems and for the determination of the degree of comfort inside the rooms in the absence of air conditioning. A methodology allowing a very simplified yet precise enough calculation of the thermal load of a room and of the inside temperature in the absence of air conditioning during periodic conditions if offered (PTBE method).

3.1 Periodic transfer functions

The behaviour of any physic system can be described by a transfer function K relating to the output $O(\tau)$ with the input $H(\tau)$:

$$O(\tau) = K * H(\tau) \qquad (34)$$

where a suitable convolution is symbolized by *.

In this case the system is a room, i.e. some walls enclosing a volume of air, while the input parameters are heat gains due to:

- -heat conduction through the envelope
- -solar radiation incoming through glass, and the radiation component of possible internal heat sources (lighting, people etc.)
- -internal convective heat sources.

Every acting quantity can be assumed to contribute to the room's thermal load by means of a transfer function the determination of which is possible by observing that, as per eq. (34), it is defined as responses $O_u(\tau)$ to a single unit impulse $H_u(\tau)$. It may be convenient not to treat as continuous functions the examined quantities, but to con-

sider a suitable sampling (time series) at given Δ time intervals /9/.
So if K_j (j=1,..∞) is the time series of O_u (τ), the output at any k-th instant, to an input H (τ), can be expressed as:

$$O_k = \sum_{1}^{\infty}{}_j K_j H_{(k-j+1)\Delta} \quad (k = 1,\dots\infty) \qquad (35)$$

since all the previous values of the excitation play a part in any specific value of the output. When in periodic conditions, an expression like eq. (35) is used for determining any of the thermal loads components, it is necessary to adopt an iterative procedure. This calls for a complex calculation and hence the need of a computer. This drawback is avoided by introducing periodic functions. They express the response of the system when an impulse repeating indefinitely at exactly the same interval of time takes place, this condition being typical of the periodic state. In practice, by adopting these new functions there is no need of following in the calculations the iterative procedure necessary when the previously mentioned transfer functions are applied. In fact, if P is the time period, and n the number of intervals Δ in the time considered, (in this work Δ= 1h, n = 24, but the methodology is rather standard), the system's response in any k-th instant to a series of impulses H_j (j=1,....n), if X_j(j=1,....n) is the periodic transfer function, is:

$$O_k = \sum_{1}^{n}{}_j X_j H_h \quad (k=1,\dots n) \qquad (36)$$

with h=k-j+1 if k-j+1 > 0, or h=n+(k-j+1) if k-j+1 ⩽ 0.

By applying a modified version of BR-202A and BR-202B computer programs (NRC of Canada) the periodic transfer functions correlating the following entities have been determined:
a) thermal load with outside air temperature
b) thermal load with inside short wave radiation
c) internal air temperature with convective heat gain.

3.2 Thermal loads evaluation

The algorithms for determining the various components of the thermal load, individually shown later, are presented with these distinctions:
a) thermal conduction gains through the room's envelope
b) solar radiation through glass areas
c) heat gains due to internal energy sources
d) heat gains due to ventilation.

3.2.1 Input parameters In a room, the morphological and thermophysical characteristics of which are known, the heating or cooling loads required with a constant internal air temperature can be calculated specifying the following hourly parameters:
-outside air temperature
-radiation falling on differently oriented surfaces
-air exchanges
-value of all heat gains due to internal energy sources
-portion of surface shadowed by external shading devices .

3.2.2 Thermal loads due to heat conduction through the building's envelope The opaque or transparent walls of a room are liable to heat conduction due to the temperature gradients establishing inside them because of both the temperature difference between inside and outside air and the occasional solar radiation impinging their external surface. Both parameters can be dealt with simultaneously by means of the "sol-air" temperature (see eq. (17)). The thermal load due to heat gains through the walls, Q_W at k-th hour can therefore be stated as:

$$Q_{W,k} = f_c \sum_{i=1}^{d} U_i \, S_i \left[\sum_{j=1}^{n} b_j (t_a - t_{sa,h}) \right]_i \tag{37}$$

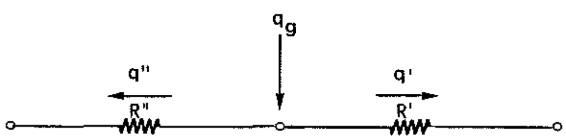
with $h=k-j+1$ if $k-j+1>0$ or $h=n+(k-j+1)$ if $k-j+1 \leqslant 0$

where:

- d: number of dispersing walls
- f_c : reduction coefficient (see eq. (9))
- b_j : periodic function values

The b-values are given in Table I as a function of the specific mass of the wall, except for very lightweight walls (including windows) the b-value of which are given in Table II as a function of the mass M of the whole structure (see eq. (28)). This parameter is very important because of its influence on mutual internal radiation exchanges.

3.2.3 Thermal load due to solar radiation falling on glass areas As already observed, the high frequency radiation (q_t) passing through a glass surface has a part in the thermal load only after having been absorbed by the internal surfaces of the building and having raised their temperatures; the component absorbed by the glass instead, is only in part let through to the room, as thermal energy (q_a), by low frequency radiation and convection, in relation to the thermophysical characteristics of that glass. Leaving aside the glass thermal capacity, the thermal radiation absorbed by a layer j-th (glass or screen) of the window ($q_{g,j}$),is instantly delivered to the inside, (q'_j),and to the outside (q''_j),in connection with the total thermal resistance (R'_j and R''_j) linking the surface to the inside and outside:



These fluxes, along with the principle of effects superposition, are not dependent from inside and outside air temperatures, and turn out to be:

$$q'_j = \frac{R''_j}{R''_j + R'_j} \, q_{g,j} \tag{38}$$

Estimating thus all the different radiation fluxes absorbed in relation to the different w elements of the window investigated, the total radiation $q_{a,k}$ transmitted to the room by the window at k-th hour, as a result of this phenomenon is:

$$q_{a,k} = \sum_{j=1}^{w} q'_{j,k} = \sum_{j=1}^{w} \frac{R''_j}{R''_j + R'_j} \, q_{g,j,k} \tag{39}$$

Let us observe that in estimating the R'_j and R''_j resistances, the values in /10/ can be used; it is particularly necessary to use a surface standard coefficient α_a on the inside surface of the window, summing up the convection and infrared radiation effects. The magnitudes of radiation transmitted and absorbed by the various surfaces in the window can be defined by resorting to the x_j($j=1,\ldots\ldots w+1$) coefficients, shown in /7/ for a wide range of cases, and stated as the ratio between the energy absorbed by the layer examined and that (J) wholly transmitted by a single strength glass (the coefficient x_{w+1}, regarding the radiation absorbed by the room, will particularly provide the energy q_t sent as radiation):

$$x_j = \frac{q_{g,j}}{J} \qquad (j=1,\ldots.w) \tag{40}$$

$$x_{w+1} = \frac{q_t}{J} \qquad (40b)$$

According to eq. (40), eq. (39) becomes:

$$q_{a,k} = \sum_{1}^{w}{}_{j} \quad \frac{R''_j}{R''_j + R'_j} \quad x_j J_k \qquad (41)$$

It can be ascertained that the x_j coefficients are not dependent on the hour chosen. The estimate of the contribution to the thermal load Q_F at k-th hour due to these two effects of solar radiation, can be obtained through the following relationship:

$$Q_{F,k} = \sum_{1}^{g}{}_{v} \; S_v \left[-f_c \sum_{1}^{n}{}_{i} \; b_i \; q_{a,h} - f_r \sum_{1}^{n}{}_{i} \; u_i \; q_{t,h} \right] \qquad (42)$$

with h=k-i+1 if k-i+1 > 0 or h=n+(k-i+1) if k-i+1 ≤ 0

where:

- g: number of windows
- S: window's sunlit area
- b_i: periodic functions, shown in tab. II, as a function of the specific mass of the structure
- u_i: periodic functions, shown in tab. III, as a function of the specific mass of the structure
- f_r: reduction coefficient given by eq. (19)

The solar radiation heat gains are very often calculated by means of the "Shading Coefficient" (ξ) /10/ defined as the ratio between the total heat gain (sum of transmitted and absorbed radiation) of the considered window and that of a single strength glass. If one wants to use this method, only the sum

$$q_{a,k} + q_{t,k} = J_k \, \xi \qquad (43)$$

can be calculated, but the subdivision of any energy gain between q_a and q_t is left to the wise choice of the designer.

3.2.4 Thermal load gains due to internal energy sources Internal energy sources quite often exist inside buildings causing thermal exchanges by convection or radiation: therefore, it is necessary in this case to determine appropriately the consequences on the thermal load, considering at k-th hour the convection component q_c as a direct contributor to the thermal load, and using for radiation fluxes q_r the relative transfer function u_i. Consequently the thermal load component Q_I, due to internal sources, can be written as:

$$Q_{I,k} = -f_r \sum_{1}^{n}{}_{j} \; q_{r,h} \; u_j \; - q_{c,k} \qquad (44)$$

with h=k-j+1 if k-j+1 > 0 or h=n+(k-j+1) if k-j+1 ≤ 0

3.2.5 Thermal load due to ventilation The presence of external air due to ventilation or to any infiltration through the envelope, gives the thermal load an additional contribution Q_V that, for the k-th instant, can be calculated as follows:

$$Q_V = \varrho c \; V_k (t_a - t_{e,k}) \qquad (45)$$

where:

V_k : volume flow rate of outdoor air

3.3 Total thermal load

The whole thermal load can be calculated, hour by hour, as the sum of the contributions shown in the paragraphs above:

$$Q_{T,k} = (Q_W + Q_F + Q_I + Q_V)_k \tag{46}$$

3.4 Inside temperature variations without climatization

It is very often necessary to know the level of the inside temperature in absence of HVAC system. This problem is particularly important as the energy crisis requires positive choices for buildings to limit heating energy consumptions (insulation values, ratio between windows and opaque walls, etc.): such architectural solutions, especially in summertime, could give rise to overheating and so make living uncomfortable.

This paragraph shows a simplified methodology that allows the calculation of internal temperature variations in absence of climatization from the knowledge of the thermal loads necessary to maintain given fixed and arbitrary conditions t'_a. The following relationship can in fact be obtained for t_a:

$$t_{a,k} = t'_a + \sum_{1}^{n}{}_i (y_i - \Delta y)\, Q_{T,h} \tag{47}$$

with $h=k-i+1$ if $k-i+1 > 0$ or $h=n+(k-i+1)$ if $k-i+1 \leqslant 0$

where:

Q_T: room's thermal load as per eq. (46) for constant internal temperature t'_a

y_i : periodic transfer function value

Δy: correction term

The y-values are given in tab. IV as a function of the specific mass M of the (see eq. (28)); the correction term Δy depends on the mean transmittance U_m (defined by eq. (10)) and is given by the relationship:

$$\Delta y = 0.2532 - 0.2350/U_m + 0.0269/U_m^2 - 0.0035/U_m^3 \tag{48}$$

and if $y_i - \Delta y \leqslant 0$ one will assume $y_i - \Delta y = 0$

3.5 Comparison

The results obtained with the above shown method are compared in figs. 6 and 7 with those of NBSLD computer program, with reference to the two rooms shown in the same figures and considering solar radiation corresponding to a cloudless sky at 21st July (latitude 40°N); the external air temperature varies between 23°C and 34°C.

4. CONCLUSION

By using only pocket or desk-top calculators, the above illustrated methods offer an accurate evaluation of some important quantities for estimating the thermal performance of buildings, i.e. total energy consumption for given periods and, on "design days", hourly thermal loads and internal temperature variations. The results obtainable are in very good agreement with those from very detailed mathematical models running on large computers; however theoretic, this agreement proves the consistency of the methods here presented along with those more sophisticated which, in their turn, have been experimentally checked with satisfactory results. By considering separately the effects of the different input parameters, the methodology used allows assessing also which of them are significant in a particular case: besides being basic for design considerations, this assessment permits the calculation itself to be performed at different complexity levels.

NOMENCLATURE

a	absorption coefficient	(–)
b	periodic conduction transfer function	(–)
c	specific heat	(J/kgK)
C	energy consumption	(J)
D	degree-days	(K)
E	amount of energy	(J)
f	reduction coefficient	(–)
H	input	(–)
I	solar radiation	(J/m^2)
K	transfer function	(–)
M	specific mass of the structure	(kg/m^2)
N	number of days	(–)
p	specific mass of the wall	(kg/m^2)
P	time span, period	(s)
P_o	response time	(s)
q	heat flux	(W/m^2)
Q	thermal load	(W)
R	thermal resistance	(m^2K/W)
s	thickness	(m)
S	area	(m^2)
t	temperature	(K)
T	day span	(s)
u	periodic radiation transfer function	(–)
U	overall thermal transmittance	(W/m^2K)
v	mean wind velocity	(m/s)
V	flow rate of exterior air	(m^3/s)
W	internal energy sources	(J)
X	periodic transfer function	(–)
y	periodic internal temperature transfer function	(K/W)
J	solar heat gain through a single strength glass	(W/m^2)
α	surface heat transfer coefficient	(W/m^2K)
η	efficiency	(–)
Θ	total "energy requirement"	(J)
λ	thermal conductivity	(W/mK)
Π	total "free energy"	(J)
ϱ	density	(kg/m^3)
τ	time	(s)

SUBSCRIPTS

a	inside
c	convection
C	cooling
e	outside
f	floor
F	radiation through the windows
g	absorbed
H	heating
I	internal energy sources
m	mean
M	long term temperature variation
max	maximum
min	minimum
r	radiation
R	radiation absorbed by the external surfaces

S	long term radiation variation
sa	sol-air
t	transmitted
T	total
V	ventilation
w	wall
W	conduction through the envelope

REFERENCES

/1/ Kusuda, T., "NBSLD, The Computer Program for Heating and Cooling Loads in Buildings", NBS, Building Science Series 69, 1976.

/2/ ASHRAE, Guide and Data Book 1970, American Society of Heating Refrigeration and Air-Conditioning Engineers, New York, 1970, Chap. 40.

/3/ Ito, N. and Kimura,K., "Convection Heat Transfer Coefficient at the Exterior Surface of Buildings Exposed to Natural Wind", TRANSACTION OF JAPANESE SOCIETY OF ARCHITECTURAL ENGINEERING, 1968.

/4/ Agnoletto, L.; Brunello, P.; Fornasieri, R.; Trapanese, G. and Zecchin, R., "Scambi Termici per radiazione tra le superfici di ambienti riscaldati: influenza sui carichi termici e sulle condizioni di benessere", (Mutual radiation heat exchanges in heated rooms, their influence on heating loads and on thermal comfort), CONDIZIONAMENTO DELL'ARIA, October 1978.

/5/ Bondi,P.; Di Filippo,P. and Reale,G., "Thermal Performance of Walls, PEG EDITRICE, 1978.

/6/ Agnoletto,L.; Brunello, P.; Vindigni, S. and Zecchin R. "Evaluation of Solar Radiation Falling on Differently Oriented Surfaces", ISTITUTO DI FISICA TECNICA - QUADERNO N. 100, Padova, October 1979.

/7/ Agnoletto,L.; Brunello,P. and Zecchin, R., "Valutazione degli apporti di energia attraverso vari tipi di superfici vetrate", (Evaluation of solar heat gains through different types of windows), ISTITUTO DI FISICA TECNICA - QUADERNO N. 95, Padova, September 1979.

/8/ Agnoletto,L., Brunello, P., Trapanese,G. and Zecchin, R., "Influenza dei parametri costruttivi sul comportamento termico degli edifici: la posizione dell'isolamento",(Influence of construction parameters on the thermal behaviour of buildings: the position of insulation), CONDIZIONAMENTO DELL'ARIA, May 1979.

/9/ Stephenson, D.G. and Mitalas, G.P., "Cooling Load Calculation by Thermal Response Factor Method", ASHRAE TRANSACTIONS, 73, 1967.

/10/ ASHRAE, Handbook of Fundamentals 1977, American Society of Heating Refrigerating and Air-Conditioning Engineers, New York, 1977, Chap. 26.

p (kg/m²) \ h	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
80	.018	.098	.140	.130	.105	.082	.065	.052	.043	.036	.031	.027	.024	.021	.019	.017	.015	.014	.013	.012	.011	.010	.009	.008
150	.012	.026	.057	.079	.086	.087	.083	.074	.065	.058	.052	.046	.040	.035	.032	.028	.025	.022	.020	.018	.016	.014	.013	.012
300	.024	.024	.025	.027	.044	.049	.055	.058	.060	.058	.056	.055	.053	.050	.048	.045	.043	.039	.037	.034	.032	.030	.028	.022
500	.033	.032	.031	.032	.035	.038	.042	.045	.047	.048	.049	.050	.051	.049	.048	.047	.046	.045	.043	.041	.039	.038	.036	.035
800	.038	.037	.036	.036	.036	.036	.038	.040	.042	.044	.045	.046	.047	.047	.046	.046	.046	.045	.044	.043	.042	.041	.040	.039

TAB. I : values of Conduction Transfer Functions, depending on the mass of the wall ($p \geq 80$ kg/m^2).

M (kg/m²) \ h	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
150	.609	.124	.083	.057	.040	.027	.018	.013	.009	.006	.004	.003	.002	.001	.001	.001	.0	.0	.0	.0	.0	.0	.0	.0
300	.507	.508	.044	.037	.032	.030	.028	.026	.024	.022	.020	.019	.018	.017	.016	.015	.014	.013	.012	.011	.010	.009	.009	.009
500	.504	.056	.043	.036	.031	.029	.027	.025	.023	.021	.020	.019	.018	.017	.016	.015	.014	.014	.013	.013	.012	.012	.011	.011
800	.502	.055	.042	.035	.031	.028	.026	.024	.022	.021	.020	.019	.018	.017	.016	.015	.015	.015	.014	.014	.014	.013	.013	.013

TAB. II: values of Conduction Transfer Functions for very lightweight walls ($p < 80$ kg/m^2), depending on the mass of the structure.

M (kg/m²) \ h	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
150	.314	.172	.119	.089	.068	.052	.041	.032	.025	.019	.015	.012	.009	.007	.006	.005	.004	.003	.002	.002	.001	.001	.001	.001
300	.232	.132	.098	.079	.065	.056	.048	.042	.036	.032	.028	.025	.022	.019	.017	.015	.012	.009	.008	.007	.006	.005	.004	.003
500	.156	.092	.072	.061	.055	.051	.047	.044	.041	.038	.036	.034	.032	.030	.028	.026	.024	.022	.021	.020	.019	.018	.017	.016
800	.138	.083	.067	.057	.052	.048	.045	.042	.040	.038	.036	.034	.032	.031	.030	.029	.028	.027	.026	.025	.024	.023	.023	.022

TAB. III : values of Radiation Transfer Functions , depending on the mass of the structure.

M (kg/m²) \ h	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
150	1.629	.386	.365	.331	.314	.297	.283	.266	.253	.239	.229	.218	.208	.198	.188	.177	.167	.161	.154	.147	.140	.133	.126	.119
300	1.468	.324	.300	.280	.266	.256	.249	.242	.236	.232	.229	.225	.222	.218	.215	.211	.208	.205	.201	.198	.195	.191	.188	.186
500	1.400	.283	.269	.263	.256	.252	.246	.242	.239	.236	.232	.229	.225	.222	.220	.218	.216	.215	.214	.213	.212	.211	.210	.209
800	1.365	.273	.266	.260	.253	.249	.245	.242	.239	.236	.234	.232	.230	.228	.227	.226	.225	.224	.223	.222	.222	.222	.221	.221

TAB. IV : values of Temperature Transfer Functions (expressed in 10^3 K/W), depending on the mass of the structure.

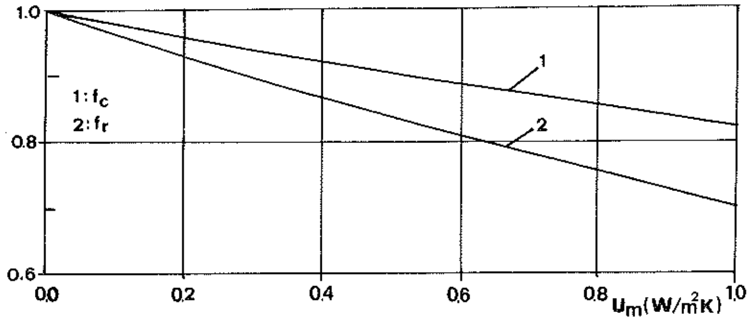


Fig. 1 : f_c and f_r coefficients.

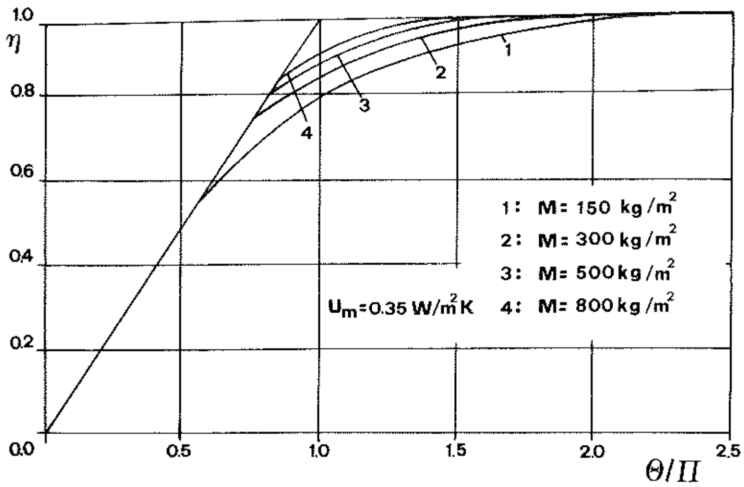


Fig. 2 : η coefficient for U_m= 0.35 W/m^{2}K

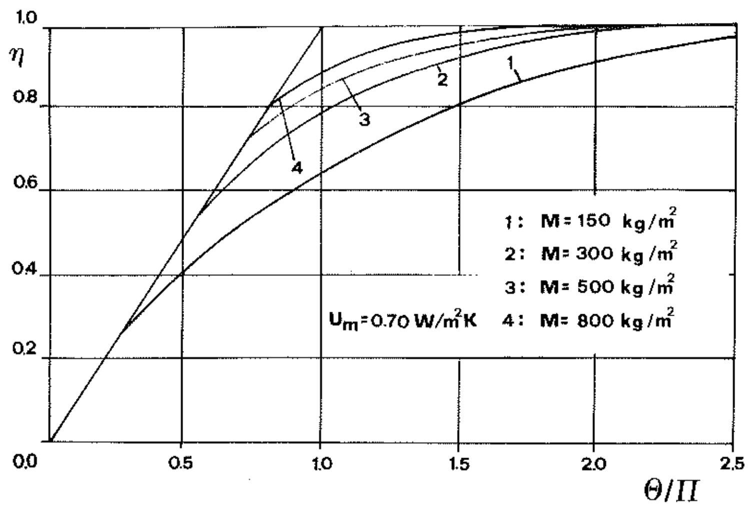


Fig. 3 : η coefficient for U_m = 0.70 W/m^{2}K

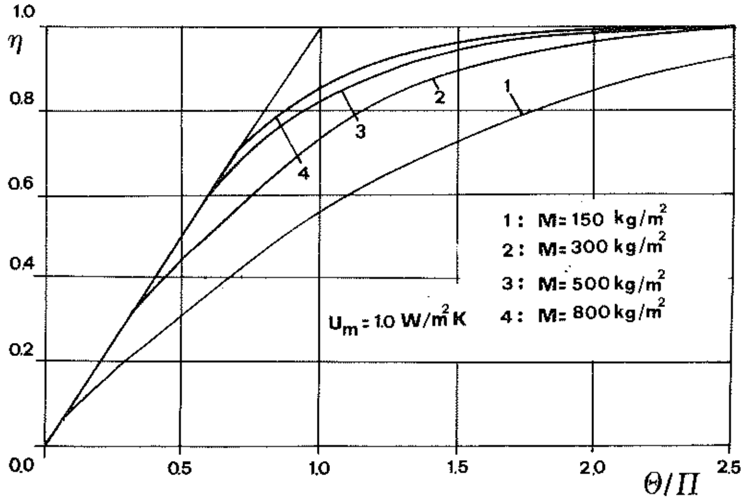


Fig. 4 : η coefficient for U_m = 1.00 W/m^{2}K

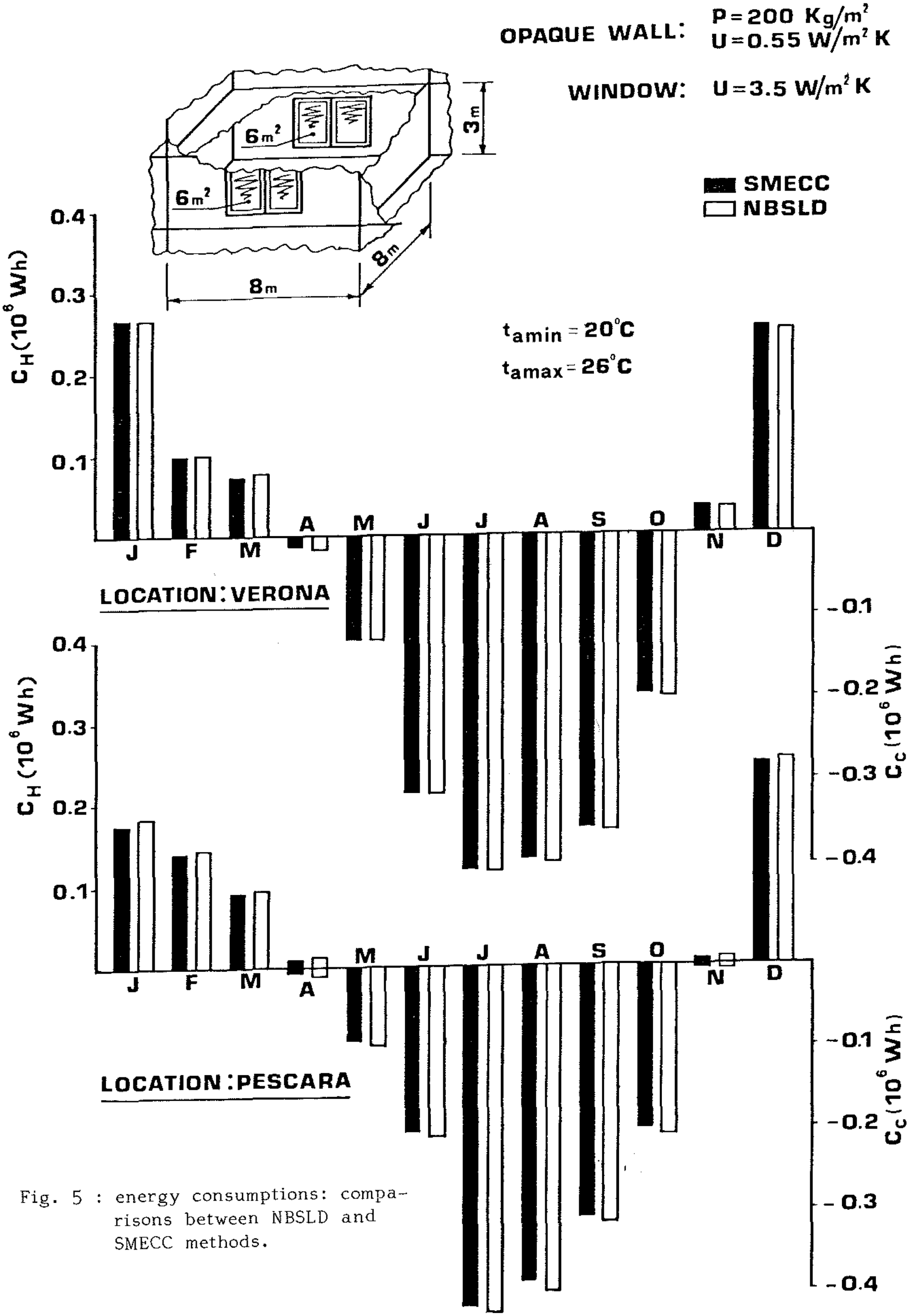


Fig. 5 : energy consumptions: comparisons between NBSLD and SMECC methods.

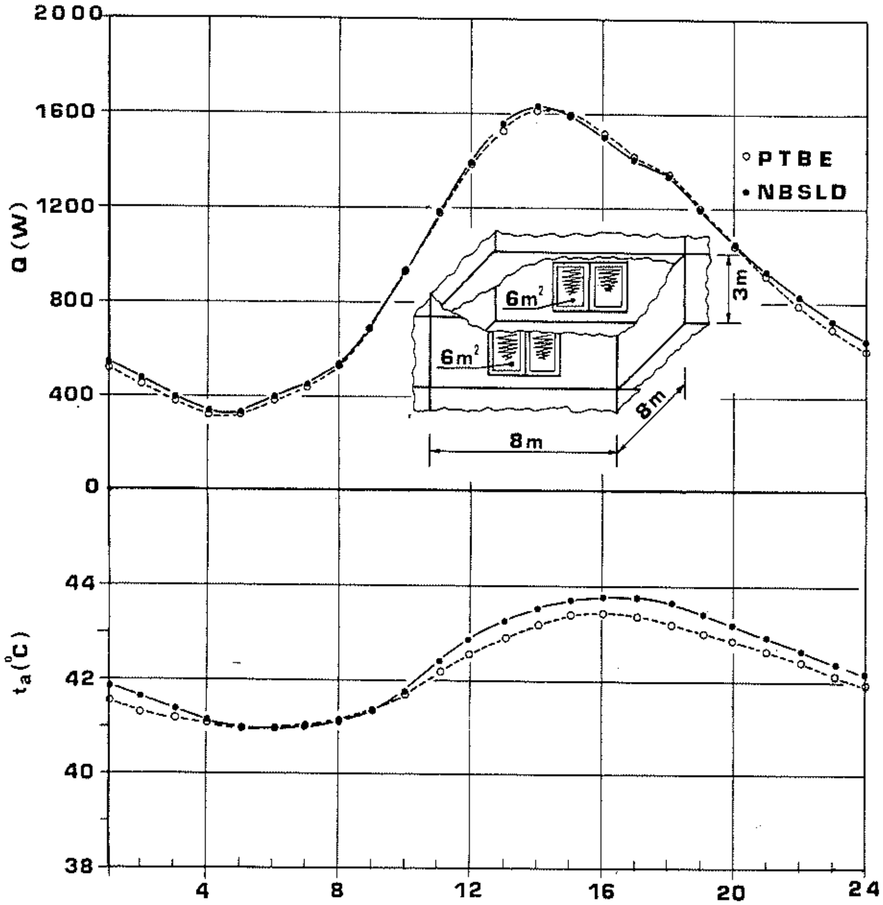


Fig. 6 : cooling load and room temperature without HVAC System: comparison between NBSLD and PTBE methods.

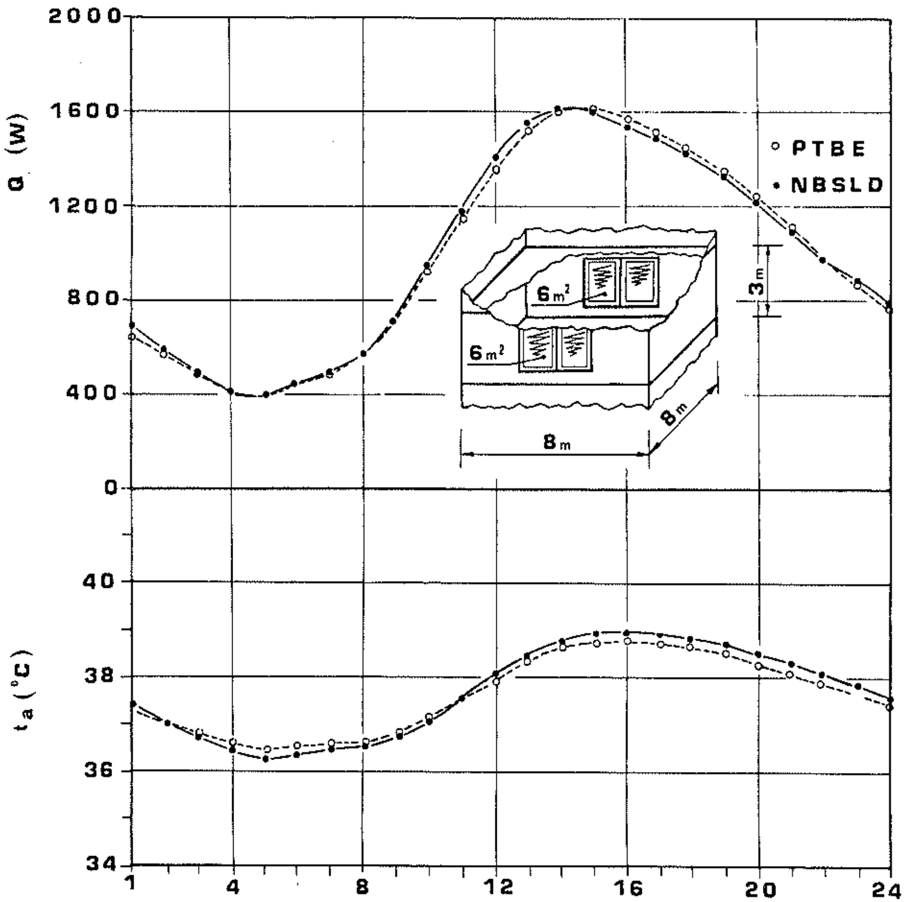


Fig. 7 : cooling load and room temperature without HVAC System: comparison between NBSLD and PTBE methods.